Bose–Einstein condensation in a disordered system: the Feynman path integral approach

2008 J. Phys. A: Math. Theor. 41 185002

(http://iopscience.iop.org/1751-8121/41/18/185002)

J. Phys. A: Math. Theor. **41** (2008) 185002 (10pp) doi:10.1088/1751-8113/41/18/185002

Bose–Einstein condensation in a disordered system: the Feynman path integral approach

Wattana Lim, Virulh Sa-yakanit and Wichit Sritrakool

Center of Excellence in Forum for Theoretical Science, Department of Physics,
Faculty of Science, Chulalongkorn University, Bangkok 10330, Thailand

Received 14 December 2007, in final form 11 March 2008
Published 18 April 2008
Online at stacks.iop.org/JPhysA/41/185002

Abstract

A modeled Bose system consisting of N particles with two-body interaction confined within volume V under the inhomogeneity of the system is investigated using the Feynman path integral approach. The two-body interaction energy is assumed to be dependent on the two-parameter interacting strength a and the correlation length l. The inhomogeneity of the system or the porosity can be represented as density $\overline{n}$ with interacting strength b and correlation length L. The mean-field approximation on the two-body interaction in the Feynman path integrals representation is performed to obtain the one-body interaction. This approximation is equivalent to the Hartree approximation in the many-body electron gas problem. This approximation has shown that the calculation can be reduced to the effective one-body propagator. Performing the variational calculations, we obtain analytical results of the ground-state energy and the condensate density which are in agreement with that from Bugoliubov's approach.

PACS numbers: 05.30.Jp, 31.15.xk

1. Introduction

Recently, Bose–Einstein condensation in a disordered system has attracted many researchers, both theorists and experimentalists [1–4]. In experiments, the systems consisting of liquid ^{4}He adsorbed in porous media, such as Vycor or silica gel, exhibit many interesting properties, which have not yet been fully understood theoretically, such as the suppression of superfluidity [5], a rich variety of elementary excitations [6], the critical behavior near the phase transition different from the bulk [7], and the condensate–noncondensate interaction generated by the inhomogeneity of the matter.

In a dilute Bose gas, the transition temperature T_c is an increasing function of the interaction parameter, $a^3\overline{N}$ [8–12] where a is the hard sphere diameter and $\overline{N}$ is the particle density. In the case of liquid ^{4}He, the transition temperature is reduced as a consequence of interparticle interactions. In the low-density regime, the critical temperature increases by the effect of the Vycor system [13]. The dilute Bose gas in the presence of quenched impurities can be worked out analytically within the Bogoliubov model by treating the random external potential as a perturbation. The effect of disorder on the ground-state energy, superfluid and condensate fraction are calculated by Astrakharchik *et al* [14]. They have found that the superfluid and condensate components of the system are suppressed by the disorder.

Huang and Meng [15] proposed a model for the three-dimensional dilute Bose gas in a random potential. They formulated the random potential for porous material with the delta-function impurity potential and then analyzed their model using the Bogoliubov transformation and taking the ensemble average. They found that both BEC and superfluidity are depressed by the random potential. They also found that the superfluidity disappears below the critical density even at 0 K and predicted that the superfluid phase enters the normal phase with decreasing temperature. However, the random potential of their model does not include the pore size and thus it is difficult to compare quantitatively with the experiment. Recently, Kobayashi and Tsubota [16] had proposed a model to improve Huang and Meng's model by adding the pore size dependence of the random potential. Their model works well, and leads to specific heat calculation.

In this paper, we consider a system of N bosons confined within the volume V. The two-body interaction is assumed to be a Gaussian function with interacting strength a and the correlation length l. The porosity of the system or the inhomogeneity is represented by density of porosity $\overline{n}$ with interacting strength b and correlation length L. The main idea of this approach is to perform the mean-field approximation on the two-body interaction in the Feynman path integral representation. This approximation is equivalent to replacing the two-body interaction into a one-body interaction with the effective random potential. As in the electron gas problem [17] we assume that the static variables are distributed randomly throughout the volume V as the inhomogeneity of the system. We will show that this effective one-body propagator allows us to determine many physical properties such as the ground-state energy, the effective mass and the condensate density.

As a consequence of the above assumptions, we will show that physical quantities can be explained using some dimensionless parameters, namely $a^3\overline{N}$ the gas parameter, $\chi \equiv n/N$ the ratio of concentration of the porosity and interacting particle, and $R \equiv \chi(b/a)^2$ the strength of disorder. The outline of this paper is as follows. In section 2, we present the model Lagrangian of the system and introduce the non-local harmonic trial action. In section 3, we present the effective one-body propagator which leads to calculating the ground-state energy in both short and long correlation lengths. In section 4, we present the statistical properties of the Bose disordered systems. The final section is devoted to the discussion and conclusion.

2. The model Lagrangian

We consider a system with N bosons interacting via the pair potentials $u(\vec{r}_i - \vec{r}_j)$ under the influence of n external impurity potentials $v(\vec{r}_i - \vec{R}_k)$ distributed randomly. The Lagrangian of the system is

$$L = \sum_{i=1}^{N} \frac{1}{2} m \dot{\vec{r}}_i^{\,2} - \sum_{i<j}^{N} u(\vec{r}_i - \vec{r}_j) - \sum_{i=1}^{N} \sum_{k=1}^{n} v(\vec{r}_i - \vec{R}_k), \tag{1}$$

where $\vec{R}_k$ are the impurity positions assumed to be completely random, and m is the mass of the bosons. In terms of Feynman's path integral representation, we can write the propagator associated with this Lagrangian as

$$P(\vec{r}_N(t), \vec{r}_N(0), t; \{\vec{R}_n\}) = \int_{\vec{r}_N(0)}^{\vec{r}_N(t)} D^N(\vec{r}_N(\tau)) \exp\left[\sum_{i=1}^{N} \frac{\mathrm{i}}{\hbar} \int_0^t \mathrm{d}\tau \left(\frac{1}{2} m \dot{\vec{r}}_i^{\,2}(\tau)\right)\right]$$
$$\times \exp\left[-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau \sum_{i<j}^{N} u(\vec{r}_i - \vec{r}_j) - \frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau \sum_{i=1}^{N}\sum_{k=1}^{n} v(\vec{r}_i - \vec{R}_k)\right]. \quad (2)$$

Edwards and Gulyaev [18] have pointed out that the average over all configurations of equation (2) can be performed exactly, and the result is

$$P(\vec{r}_N(t), \vec{r}_N(0), t) = \int_{\vec{r}_N(0)}^{\vec{r}_N(t)} D^N(\vec{r}_N(\tau)) \exp\left[\sum_{i=1}^{N} \frac{\mathrm{i}}{\hbar} \int_0^t \mathrm{d}\tau \left(\frac{1}{2} m \dot{\vec{r}}_i^{\,2}(\tau)\right)\right]$$
$$\times \exp\left[-\sum_{i<j}^{N} \frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\, u(\vec{r}_i(\tau) - \vec{r}_j(\tau)) + \sum_{i=1}^{N} \bar{n} \int \mathrm{d}\vec{R}_k \left(\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\, v(\vec{r}_i(\tau) - \vec{R}_k)} - 1\right)\right], \quad (3)$$

where $\bar{n} \equiv n/V$ is the number density of the impurity. To perform the Gaussian approximation of the propagator, we expand the exponential of the impurity potential v in equation (3) and keep terms only to the second order of v. The first-order term then gives the average of scattering potential, while the second-order term leads to the fluctuations of the scattering potential. Alternatively, we may think of the scattering potential as a random potential, and the result of Edwards and Gulyaev enables us to explicitly calculate its average and fluctuations. In detail, expanding the exponential leads to

$$\sum_{i=1}^{N} \bar{n} \int \mathrm{d}\vec{R}_k \left(\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\, v(\vec{r}_i(\tau) - \vec{R}_k)} - 1\right)$$
$$\approx -\sum_{i=1}^{N} \frac{\mathrm{i}}{\hbar}\bar{n} \int_0^t \mathrm{d}\tau\, V(\tau) + \sum_{i=1}^{N} \left(-\frac{\mathrm{i}}{\hbar}\right)^2 \frac{\bar{n}}{2} \int_0^t \int_0^t \mathrm{d}\tau\, \mathrm{d}\sigma\, W_L(\vec{r}_i(\tau) - \vec{r}_i(\sigma)). \quad (4)$$

For the impurity potential in the form of a Gaussian function,

$$\bar{n} v(\vec{r}(\tau) - \vec{R}_k) = \frac{2\bar{n}\pi\hbar^2 b}{m} (\pi L^2)^{-3/2} \exp\left[-\left(\frac{\vec{r}(\tau) - \vec{R}_k}{L}\right)^2\right], \quad (5)$$

the mean potential is

$$\bar{n} V(\tau) = \bar{n} \int \mathrm{d}\vec{R}_k v\left(\vec{r}_i(\tau) - \vec{R}_k\right) = \frac{\xi_b}{2} \quad (6)$$

and the correlation takes the form

$$\frac{\bar{n}}{2} W_L(\vec{r}_i(\tau) - \vec{r}_i(\sigma)) = \frac{\bar{n}}{2} \int \mathrm{d}\vec{R}_k v(\vec{r}_i(\tau) - \vec{R}_k) v(\vec{r}_i(\sigma) - \vec{R}_k)$$
$$= \xi_L \exp\left[-\frac{(\vec{r}_i(\tau) - \vec{r}_i(\sigma))^2}{2L^2}\right], \quad (7)$$

where b and L, respectively, represent the interacting strength and the correlation length, $\xi_b = 4\bar{n}\pi\hbar^2 b/m$ and $\xi_L = (\bar{n}/2)(2\pi\hbar^2 b/m)^2(\pi L^2)^{-3/2}$ with the dimension of energy squared. It will be seen later that the correlation length is comparable with the pore size of the porous material as discussed by Kobayashi and Tsubota [16].

3. The effective one-body propagator

In order to compare our approach with other theories, we must first transform the two-body interaction into a one-body interaction. This can be achieved by replacing one of the dynamical variables $\vec{r}_j(\tau)$ of the two-body interaction into the one-body with the static parameter $\vec{R}_j$. We assume that the $\vec{R}_j$ distribute completely randomly. Physically, this approximation resembles the Hartree approximation in the electron gas problem. In this case, one assumes that the wavefunctions of other particles are classical. Therefore, one can replace other wavefunctions as electrostatic charges distributed throughout the solid. In the Feynman path integral approach, we work on the configuration representation instead of the wavefunction. Performing the mean-field random average, we can write the propagator as

$$P(\vec{r}_N(t), \vec{r}_N(0), t) = \int_{\vec{r}_N(0)}^{\vec{r}_N(t)} D^N(\vec{r}_N(\tau)) \exp\left[\sum_{i=1}^{N} \frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau \left(\frac{1}{2}m\dot{\vec{r}}_i^{\,2}(\tau)\right)\right]$$
$$\times \exp\left[\sum_{i=1}^{N} \frac{\overline{N}}{2}\int \mathrm{d}\vec{R}_j\left(\mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\, u(\vec{r}_i(\tau)-\vec{R}_j)} - 1\right)\right.$$
$$\left. - \frac{\mathrm{i}}{\hbar}N\frac{\xi_b}{2}t + \sum_{i=1}^{N}\left(-\frac{\mathrm{i}}{\hbar}\right)^2 \frac{\overline{n}}{2}\int_0^t\int_0^t \mathrm{d}\tau\,\mathrm{d}\sigma\, W_L(\vec{r}_i(\tau) - \vec{r}_i(\sigma))\right]. \tag{8}$$

The two-particle interacting potential is assumed to be a Gaussian function potential of the form

$$\overline{N}u(\vec{r}(\tau) - \vec{R}_j) = \frac{4\overline{N}\pi\hbar^2 a}{m}(\pi l^2)^{-3/2}\exp\left[-\left(\frac{\vec{r}(\tau) - \vec{R}_j}{l}\right)^2\right]. \tag{9}$$

The two parameters, a and l, represent the interacting strength and the correlation length of two-body interaction, respectively. Again, expanding the exponential term, we obtain the first mean-field contribution. The second term gives rise to the fluctuation of the mean-field contribution. The same method used to obtain the porosity is applied to this problem. Inserting the interacting potential, we finally have the one-body propagator. Therefore, the effective one-body propagator is

$$P(\vec{r}_N(t), \vec{r}_N(0), t) = \int_{\vec{r}_N(0)}^{\vec{r}_N(t)} D^N(\vec{r}_N(\tau)) \exp\left[\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau \sum_{i=1}^{N}\frac{1}{2}m\dot{\vec{r}}_i^{\,2}(\tau)\right]$$
$$\times \exp\left[-\frac{\mathrm{i}}{\hbar}N\frac{\xi_a}{2}t + \sum_{i=1}^{N}\left(-\frac{\mathrm{i}}{\hbar}\right)^2\frac{\overline{N}}{4}\int_0^t\int_0^t \mathrm{d}\tau\,\mathrm{d}\sigma\, W_l(\vec{r}_i(\tau) - \vec{r}_i(\sigma))\right]$$
$$\times \exp\left[-\frac{\mathrm{i}}{\hbar}N\frac{\xi_b}{2}t + \sum_{i=1}^{N}\left(-\frac{\mathrm{i}}{\hbar}\right)^2\frac{\overline{n}}{2}\int_0^t\int_0^t \mathrm{d}\tau\,\mathrm{d}\sigma\, W_L(\vec{r}_i(\tau) - \vec{r}_i(\sigma))\right], \tag{10}$$

where $\xi_a = 4\overline{N}\pi\hbar^2 a/m$ and $\xi_l = (\overline{N}/4)(4\pi\hbar^2 a/m)^2(2\pi l^2)^{-3/2}$. In order to perform the calculation, we follow the method we developed to study the electron in random potential. The main idea is to introduce a trial action with the non-local harmonic action given as

$$S_0 = \sum_{i=1}^{N}\int_0^t \mathrm{d}\tau \frac{1}{2}m\dot{\vec{r}}_i^{\,2}(\tau) - \sum_{i=1}^{N}\frac{m\omega^2}{4t}\int_0^t\int_0^t \mathrm{d}\tau\,\mathrm{d}\sigma(\vec{r}_i(\tau) - \vec{r}_i(\sigma))^2, \tag{11}$$

where ω is a variational parameter. Fortunately, this model can be solved exactly. The result is

$$P_0(\vec{r}_N(t), \vec{r}_N(0), t) = \left(\frac{m}{2\pi \mathrm{i}\hbar t}\right)^{\frac{3}{2}N} \left(\frac{\omega t}{2\sin\left(\frac{\omega t}{2}\right)}\right)^{3N} \times \exp\left[\frac{\mathrm{i}}{\hbar}\sum_{i=1}^{N}\frac{m\omega}{4}\cot\left(\frac{\omega t}{2}\right)(\vec{r}_i(t) - \vec{r}_i(0))^2\right]. \quad (12)$$

We can rewrite the effective one-body propagator in terms of the trial propagator as

$$P(\vec{r}_N(t), \vec{r}_N(0), t) = P_0(\vec{r}_N(t), \vec{r}_N(0), t)\left\langle \exp\left[\frac{\mathrm{i}}{\hbar}(S - S_0)\right]\right\rangle_{S_0}, \quad (13)$$

where the symbol $\langle\,\rangle_{S_0}$ represents the average with respect to the trial action S_0. Expanding the above average in terms of cumulants and keeping only the first cumulant [17], we have

$$P(\vec{r}_N(t), \vec{r}_N(0), t) \simeq P_0(\vec{r}_N(t), \vec{r}_N(0), t)\exp\left[\frac{\mathrm{i}}{\hbar}\langle S - S_0\rangle_{S_0}\right]. \quad (14)$$

The propagator P_0 is given in equation (12). The next step is to evaluate the action difference $\frac{\mathrm{i}}{\hbar}\langle S - S_0\rangle_{S_0}$ which can be written as

$$\frac{\mathrm{i}}{\hbar}\langle S - S_0\rangle_{S_0} = -\frac{\mathrm{i}}{\hbar}N\frac{\xi_a}{2}t + \left[\sum_{i=1}^{N}\left(\frac{\mathrm{i}}{\hbar}\right)^2\int_0^t\int_0^\tau \mathrm{d}\tau\,\mathrm{d}\sigma\,\xi_l\frac{1}{A_l^{3/2}(\tau,\sigma)}\exp\left[-\frac{\vec{R}^2(\tau,\sigma)}{2l^2A_l^{3/2}(\tau,\sigma)}\right]\right] - \frac{\mathrm{i}}{\hbar}N\frac{\xi_b}{2}t + \left[\sum_{i=1}^{N}\left(\frac{\mathrm{i}}{\hbar}\right)^2\int_0^t\int_0^\tau \mathrm{d}\tau\,\mathrm{d}\sigma\,\xi_L\frac{1}{A_L^{3/2}(\tau,\sigma)}\exp\left[-\frac{\vec{R}^2(\tau,\sigma)}{2L^2A_L^{3/2}(\tau,\sigma)}\right]\right] - \frac{\mathrm{i}}{\hbar}\langle S_0\rangle_{S_0}, \quad (15)$$

where

$$\vec{R}^2(\tau,\sigma) = \langle \vec{r}_i(\tau) - \vec{r}_i(\sigma)\rangle_{S_0}^2 = \left(\frac{\cos\left(\frac{\omega}{2}(t - |\tau+\sigma|)\right)\sin\left(\frac{\omega}{2}(\tau-\sigma)\right)}{\sin\left(\frac{\omega}{2}t\right)}\right)^2(\vec{r}(t) - \vec{r}(0))^2, \quad (16)$$

and

$$A_l^{3/2}(\tau,\sigma) = \left(1 + \frac{2}{l^2}G(\tau,\sigma)\right)^{3/2}; \qquad A_L^{3/2}(\tau,\sigma) = \left(1 + \frac{2}{L^2}G(\tau,\sigma)\right)^{3/2}, \quad (17)$$

where the Green function is given by

$$G(\tau,\sigma) = \frac{\mathrm{i}\hbar\sin\left(\frac{\omega(\tau-\sigma)}{2}\right)\sin\left(\frac{\omega(t-(\tau-\sigma))}{2}\right)}{m\omega\sin\left(\frac{\omega t}{2}\right)}. \quad (18)$$

The average $\langle S_0\rangle_{S_0}$ in equation (15) can be evaluated exactly. The result has been reported by one of us [17]. Collecting all contributions, we obtain the approximated propagator with the first cumulant approximation as

$$P\left(\vec{r}_N(t), \vec{r}_N(0), t\right) = \left(\frac{m}{2\pi\mathrm{i}\hbar t}\right)^{\frac{3}{2}N}\left(\frac{\omega t}{2\sin\left(\frac{\omega t}{2}\right)}\right)^{3N} \times \exp\left[\sum_{i=1}^{N}\frac{\mathrm{i}}{\hbar}m\left[\frac{1}{4}\omega t\cot\left(\frac{1}{2}\omega t\right) - \left(\frac{1}{2}\omega t\csc\left(\frac{1}{2}\omega t\right)\right)^2\right]\frac{(\vec{r}_i(t) - \vec{r}_i(0))^2}{2t}\right] \times \exp\left[\frac{3N}{2}\left(\frac{1}{2}\omega t\cot\left(\frac{1}{2}\nu t\right) - 1\right)\right]\exp\left[-\frac{\mathrm{i}}{\hbar}N\left(\frac{\xi_a}{2} + \frac{\xi_b}{2}\right)t\right]$$

$$\times \exp\left[\sum_{i=1}^{N}\left(\frac{\mathrm{i}}{\hbar}\right)^2 \int_0^t \int_0^\tau \mathrm{d}\tau\, \mathrm{d}\sigma \xi_l \frac{1}{A_l^{3/2}(\tau,\sigma)} \exp\left[-\frac{\overrightarrow{R}^2(\tau,\sigma)}{2l^2 A_l^{3/2}(\tau,\sigma)}\right]\right]$$
$$\times \exp\left[\sum_{i=1}^{N}\left(\frac{\mathrm{i}}{\hbar}\right)^2 \int_0^t \int_0^\tau \mathrm{d}\tau\, \mathrm{d}\sigma \xi_L \frac{1}{A_L^{3/2}(\tau,\sigma)} \exp\left[-\frac{\overrightarrow{R}^2(\tau,\sigma)}{2L^2 A_L^{3/2}(\tau,\sigma)}\right]\right]. \quad (19)$$

3.1. Ground-state energy

To obtain the ground-state energy, we replace t by $-\mathrm{i}\hbar\beta$. This transformation allows us to turn the propagator into the density matrix. For large β the one-body density matrix can be written as

$$\rho(\overrightarrow{r}(\beta), \overrightarrow{r}(0), \beta) \underset{\beta\to\infty}{=} \left(\frac{m^*}{2\pi\hbar^2\beta}\right)^{3/2} \exp\left[-E_0\beta - \frac{m^*}{2\hbar^2\beta}(\overrightarrow{r}(\beta) - \overrightarrow{r}(0))^2\right]. \quad (20)$$

The leading term gives the ground-state energy and the second term in the exponential gives the wavefunction of the system. To be able to use the Feynman variational principle, we take the trace of the density matrix. Since the system is translational invariant, only the diagonal part contributes to the ground-state energy. The trace gives right to the volume of the system,

$$\mathrm{Tr}\,\rho(\overrightarrow{r}(\beta), \overrightarrow{r}(0), \beta) \underset{\beta\to\infty}{=} V\left(\frac{m^*}{2\pi\hbar^2\beta}\right)^{3/2} \exp[-E_0\beta], \quad (21)$$

where the ground-state energy is

$$E_0 = \frac{3}{4}E_\omega + 4\pi a^3\overline{N}E_a\left(1 + \frac{b}{a}\frac{n}{N}\right) + \frac{2\xi_l}{\theta_l^3 E_\omega}\left(1 - \theta_l + \ln\left(\frac{1}{2} + \frac{1}{2\theta_l}\right)\right)$$
$$+ \frac{2\xi_L}{\theta_L^3 E_\omega}\left(1 - \theta_L + \ln\left(\frac{1}{2} + \frac{1}{2\theta_L}\right)\right), \quad (22)$$

where $\theta_l = \sqrt{1 + 2E_l/E_\omega}, \theta_L = \sqrt{1 + 2E_L/E_\omega}$, $E_a = \hbar^2/2ma^2$, $E_l = \hbar^2/2ml^2$, $E_L = \hbar^2/2mL^2$ and $E_\omega = \hbar\omega$ is the variational parameter to be determined.

3.1.1. White noise limit. Taking the small l and L limit so that $2E_l/E_\omega \gg 1$ and $2E_L/E_\omega \gg 1$, we can write the ground-state energy in terms of the dimensionless parameters as

$$E_0 = 4\pi a^3\overline{N}E_a\left(1 + \frac{b}{a}\frac{n}{N}\right) - 4\pi a^3\overline{N}E_a\left(\sqrt{\frac{2}{\pi}}\frac{a}{l} + \frac{b}{a}\frac{n}{N}\frac{1}{\sqrt{2\pi}}\frac{b}{L}\right)$$
$$+ \frac{3}{4}E_\omega + 2\,(1 - \ln 2)\,a^3\overline{N}\sqrt{E_a}\sqrt{E_\omega}\sqrt{\pi}\,(2 + R)\,, \quad (23)$$

where $R = \chi\,(b/a)^2$ represents the strength of disorder and $\chi = n/N$. Minimizing the ground-state energy by solving $\mathrm{d}E_0/\mathrm{d}E_\omega = 0$, we get

$$E_\omega = \left(\tfrac{4}{3}\sqrt{\pi}(1 - \ln 2)a^3\overline{N}(2 + R)\right)^2 E_a. \quad (24)$$

In order to avoid the divergency in taking the 'white noise' limit, we set $l = a$ and $L = r_p$. The physical meaning of the hard-sphere model is that the correlation length of the interacting particle and the impurity cannot be less than diameter of particle a and diameter of impurity or pore size r_p. Substituting equation (24) into equation (23), we obtain

$$\frac{E_0}{E_a} = 4\pi a^3\overline{N}\left(1 + \frac{b}{a}\frac{n}{N}\right) + 4\pi(1 - \ln 2)^2(a^3\overline{N}(2 + R))^2 - 4\pi a^3\overline{N}\left(\sqrt{\frac{2}{\pi}} + \frac{R}{\sqrt{2\pi}}\frac{a}{r_p}\right). \quad (25)$$

We note that for the white noise limit, a and b are scattering lengths of the interacting particles and impurities. This result can be compared with the result of Kobayashi and Tsubota [16] for the case of $l \to 0$ and $L = r_p$ as

$$\frac{E_0}{E_a} = 4\pi a^3\overline{N}\left(1 + \frac{b}{a}\frac{n}{N}\right) + (a^3\overline{N})^{3/2}\frac{512\sqrt{\pi}}{15} + 2(a^3\overline{N})^{3/2}\pi^{3/2}R\left[-\mathrm{e}^{2\alpha}(5+4\alpha)\{1 - \operatorname{erf}\sqrt{2\alpha}\} + \sqrt{\frac{2}{\pi\alpha}}(1+\alpha)\right], \tag{26}$$

where $\alpha = (a^3\overline{N}/\pi)\left(r_p^2/a^2\right)$. By examining equations (25) and (26), one finds that the first term in both approaches arises from the Bose gases without the disorder. To include the disorder, we find that both approaches are slightly different due to different approaches. For the case of Kobayashi and Tsubota, the approach is based on the extension of Huang and Meng theory [15] for finite correlation length. The method of Huang and Meng is the generalization of the Bogolubov model which is represented in the momentum representation. In our approach, we start with the Feyman path integral approach which is written in the configuration representation. In principle, both approaches should give the same result if there is no approximation. Our result can be compared with the result of Astrakharchik *et al* [14] for the case of both the two-body interaction and the random interaction with the delta potential functions

$$\frac{E_0}{E_a} = 4\pi a^3\overline{N}\left(1 + \frac{b}{a}\frac{n}{N}\right) + (a^3\overline{N})^{3/2}\left(\frac{512\sqrt{\pi}}{15} + 16\pi^{3/2}R\right). \tag{27}$$

3.1.2. Long length limit. We take the large l and L limit so that $2E_l/E_\omega \ll 1$ and $2E_L/E_\omega \ll 1$. Therefore we can expand equation (22) in the power of $2E_l/E_\omega$ and $2E_L/E_\omega$ as $\theta_l \simeq 1 + E_l/E_\omega$ and $\ln(1/2 + 1/2\theta_l) \simeq -E_l/2E_\omega$. Thus we can write equation (22) as

$$E_0 = \frac{3}{4}E_\omega + 4\pi a^3\overline{N}E_a\left(1 + \frac{b}{a}\frac{n}{N}\right) - \frac{12a^3\overline{N}\sqrt{E_a}\sqrt{2\pi}E_l^{5/2}}{E_\omega^2} - \frac{6a^3\overline{N}\sqrt{E_a}\sqrt{2\pi}E_L^{5/2}R}{E_\omega^2}. \tag{28}$$

Minimizing the ground-state energy by solving $\mathrm{d}E_0/\mathrm{d}E_\omega = 0$, we get

$$E_\omega = -2\sqrt{2}\left(a^3\overline{N}\sqrt{\pi E_a}\left(2E_l^{5/2} + E_L^{5/2}R\right)\right)^{1/3}. \tag{29}$$

Substituting equation (29) into equation (28), we obtain

$$\frac{E_0}{E_a} = 4\pi a^3\overline{N}\left(1 + \frac{b}{a}\frac{n}{N}\right) - \frac{9}{2\sqrt{2}}\left(\sqrt{\pi}a^3\overline{N}\left(\frac{a}{l}\right)^5\left(2 + \left(\frac{l}{L}\right)^5 R\right)\right)^{1/3}. \tag{30}$$

We find that the second term in equation (30) is very small. Therefore, we can approximate the ground state energy for the long length limit as the mean-field energy.

4. Quantum statistics

The propagator of the system is reduced to the free particle propagator when $\omega = 0$. This means that we measure the system using the free particle propagator. Taking the limit $\omega = 0$, we have

$$\langle \vec{r}_i(\tau) - \vec{r}_i(\sigma)\rangle_{S_0} = \frac{(\tau - \sigma)}{2}(\vec{r}(t) - \vec{r}(0)) \tag{31}$$

and

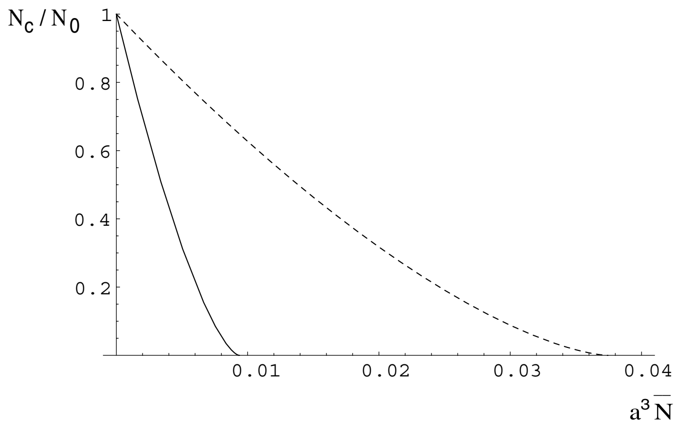

Figure 1. The condensate density against $a^3\overline{N}$. The dash line is condensate density without impurity and the solid line is condensate density in the presence of the impurity for the white noise limit. ($l = a$, $L/a = 3$, $n/N = 1/2$, $b/a = 2$ and $R = 2$.)

$$G(\tau,\sigma) = \frac{\mathrm{i}\hbar(\tau-\sigma)(t-(\tau-\sigma))}{2mt}. \tag{32}$$

Substituting equations (31) and (32) into equation (19) and expanding the exponential term up to the second order, we can calculate the first and the second terms exactly. For the large-t limit, the propagator can be written in a simple form as

$$P(\vec{r}(t),\vec{r}(0),t) = \left(\frac{m^*}{2\pi\mathrm{i}\hbar t}\right)^{3N/2} \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}NE_0 t} \exp\left[\frac{\mathrm{i}}{\hbar}\frac{m^*}{2t}\sum_{i=1}^{N}(\vec{r}_i(t)-\vec{r}_i(0))^2\right], \tag{33}$$

where the effective mass is

$$m^* = m\left(1 - \frac{16\sqrt{2\pi}}{3}a^3\overline{N}\left(\frac{2l}{a}+\frac{L}{a}R\right)\right), \tag{34}$$

and the ground-state energy is

$$E_0 = 4\pi a^3\overline{N}E_a\left(1+\frac{b}{a}\frac{n}{N}\right) - 4\pi a^3\overline{N}E_a\sqrt{\frac{2}{\pi}}\frac{a}{l} - 4\pi a^3\overline{N}E_a\frac{b}{a}\frac{n}{N}\frac{1}{\sqrt{2\pi}}\frac{b}{L}. \tag{35}$$

This ground-state energy is the same as that in equation (23) in the limit of $E_\omega \to 0$. The effective mass ratio is

$$\frac{m^*}{m} = 1 - \frac{16\sqrt{2\pi}}{3}a^3\overline{N}\left(\frac{2l}{a}+\frac{L}{a}R\right). \tag{36}$$

Thus we obtain the ground-state energy and effective mass for any correlation lengths. Therefore the N-body density matrix can be written as

$$\rho(\vec{r}(\beta),\vec{r}(0),\beta) = \left(\frac{m^*}{2\pi\hbar^2\beta}\right)^{3N/2} \mathrm{e}^{-NE_0\beta} \exp\left[-\frac{m^*}{2\hbar^2\beta}\sum_{i=1}^{N}(\vec{r}_i(\beta)-\vec{r}_i(0))^2\right]. \tag{37}$$

In order to study in statistical mechanics, we sum over all possible permutations. Therefore, the partition function can be written as

$$Q^{(\mu)} = \exp\left[V\left(\frac{m^*}{2\pi\hbar^2\beta}\right)^{3/2}\zeta_{5/2}(\alpha)\right], \tag{38}$$

where $\alpha = e^{\mu\beta}$, μ is a chemical potential and $\zeta_{5/2}(\alpha) = \sum_{\nu=1}^{\infty} \frac{\alpha^\nu}{\nu^{5/2}}$. Using the definition $\overline{N}_c = \frac{1}{\beta Q^{(\mu)}} \frac{\partial Q^{(\mu)}}{\partial \mu}$, we obtain the condensate density as

$$\overline{N}_c = \overline{N}_0 \left(1 - \frac{16\sqrt{2\pi}}{3} a^3 \overline{N} \left(\frac{2l}{a} + \frac{L}{a} R \right) \right)^{3/2}, \tag{39}$$

where the condensate density $\overline{N}_0 = \left(\frac{mkT_c}{2\pi\hbar^2}\right)^{3/2} \zeta_{3/2}(1)$ for ideal gas at $T < T_c$. We find that the condensate density is depleted by the repulsive interaction and the strength of disorder (see figure 1). If there is no interaction and impurity, the system is only an ideal Bose gas.

5. Conclusion

In this paper, we consider Bose systems in a disordered system with a finite correlation length L and with a pair potential of interacting particles with correlation length l using the Feynman path integral approach. The two-body interacting strength is taken as a and the impurity is taken as b. For the white noise limit both a and b are scattering lengths. The main idea of this approach is to perform the mean-field approximation in the Feynman path integrals. We replace one of the dynamic variables of the two-body interaction into a static parameter and assume that the static parameters are completely random distribution throughout the sample. This average over all configurations of the static parameters can be compared with the Hartree approximation of the many electron gas problem. The advantage of using the Feynman path integral is that we can take care of the divergence arising from using the delta potential. For the white noise limit $l \to a$ and $L = r_p$, we can compare with the result of Kobayashi and Tsubota with $l \to 0$ and $L = r_p$. For the case of $l \to 0$ and $L \to 0$, we can compare with the result of Astrakharchik *et al* for the mean-field contribution. For the random potential contribution, it is different due to different approaches. Moreover, the Feynman path integral theory can be used to study the long correlation length of interacting particles and impurities.

We have also calculated the condensate density using the one-body propagator. In the limit $\omega \to 0$, the propagator of the system reduces to that of the free particle. The partition function is obtained after summing over all permutations of the N-body density matrix. We have found that for dilute gas, the repulsive interacting particles, the strength of disorder and the correlation length of both interacting particles and impurities suppress the condensate density.

Acknowledgments

The authors would like to thank Professor Dr Henry R Glyde of Department of Physics and Astronomy, University of Delaware, USA for his valuable advice and discussions. One of the authors (Wattana Lim) also wishes to thank the financial support from the Royal Golden Jubilee PhD Program of the Thailand Research Fund (TRF).

References

[1] Yukalov V I and Graham R 2007 *Phys. Rev.* A **75** 023619
[2] Sanchez-Palencia L, Clément D, Lugan P, Bouyer P, Shlyapnikov G V and Aspect A 2007 *Phys. Rev. Lett.* **98** 210401
[3] Paul T, Schlagheck P, Leboeuf P and Pavloff N 2007 *Phys. Rev. Lett.* **98** 210602
[4] Lugan P, Clément D, Bouyer P, Aspect A, Lewenstein M and Sanchez-Palencia L 2007 *Phys. Rev. Lett.* **98** 170403

[5] Reppy John D 1992 *J. Low Temp. Phys.* **87** 205
[6] Singh Kanwal G and Rokhar Danial S 1994 *Phys. Rev.* B **49** 9013
[7] Wong G F S, Crowell P A, Cho H A and Reppy J D 1993 *Phys. Rev.* B **48** 3858
[8] Grüter P and Ceperley D 1997 *Phy. Rev. Lett.* **79** 3549
[9] Stoof H T C 1992 *Phys. Rev.* A **45** 8398
[10] Holzmann M, Grüter P and Laloë F 1999 *Euro. Phys. J.* B **10** 739
[11] Baym G, Blaizot J-P and Holzmann M 1999 *Phys. Rev. Lett.* **83** 1703
[12] Holzmann M and Krauth W 1999 *Phys. Rev. Lett.* **83** 2687
[13] Reppy J D, Crooker B C, Hebral B, Corwin A D, He J and Zassenhaus G M 2000 *Phys. Rev. Lett.* **84** 2060
[14] Astrakharchik G E, Boronat J, Casulleras J and Giorgini S 2002 *Phys. Rev.* A **66** 023603–2
[15] Huang K and Meng H F 1992 *Phys. Rev. Lett.* **69** 644
[16] Kobayashi M and Tsubota M 2002 *Phys. Rev.* B **66** 174516
[17] Sa-yakanit V 1974 *J. Phys. C: Solid State Phys.* **7** 2849
[18] Edwards S F and Gulyaev V B 1964 *Proc. Phys. Soc.* **83** 495